

POSTOTCI

$$P = \frac{p}{100} \cdot G$$

1.

G - glavica, P - postotni iznos, p - postotak

① Koliko je

a) 20% od 30 $\Rightarrow \frac{20}{100} \cdot 30 = \frac{20 \cdot 30}{100} = 6$

(p = 20, G = 30)

b) 13% od 40 \Rightarrow

c) 85% od 620 \Rightarrow

d) 57% od 1200 \Rightarrow

② Od kojeg broja

a) 30% iznosi 12 $\Rightarrow 12 = \frac{30}{100} \cdot G \Rightarrow G = \frac{12}{0.3} = 40$

(p = 30, P = 12)

(12 = 0.3 \cdot G) ili G = \frac{12 \cdot 100}{3} = 40

b) 35% iznosi 280 \Rightarrow

c) 80% iznosi 200 \Rightarrow

d) 64% iznosi 332.8 \Rightarrow

③ Koliki postotak broja

a) 72 iznosi 24 $\Rightarrow 24 = \frac{p}{100} \cdot 72 \Rightarrow p = \frac{24 \cdot 100}{72} = 33.33\%$

(G = 72, P = 24)

b) 240 iznosi 72 \Rightarrow

c) 380 iznosi 95 \Rightarrow

d) 185 iznosi 151.7 \Rightarrow

③ Roba košta 220 kn.

a) Kolika je njena cijena ako ju smanjimo 10%

$$220 - 220 \cdot \frac{10}{100} = 220 - 22 = 198 \text{ kn}$$

ili ostaje 90% od 220 kn, a to je $220 \cdot \frac{90}{100} = 220 \cdot 0.9 = 198 \text{ kn}$

b) Kolika je cijena robe ^(220kn) ako je povećamo 5%

$$220 + 220 \cdot \frac{5}{100} = 220 + 11 = 231 \text{ kn}$$

ili će biti 105% od 220, a to je $220 \cdot \frac{105}{100} = 220 \cdot 1.05 = 231 \text{ kn}$

⊛ Roba košta 180 kn. Koliko će koštati ako je pojeftini 12% ?

⊛ Roba košta 350 kn. Koliko će koštati ako je poskupi 8% ?

⊛ Roba košta 200 kn. Poskupila je 10%, a nakon toga pojeftinila 10%. Kolika joj je cijena?

$a^2, a^3, a^{-7}, a^{11}, \dots$ - potencije jednakih baza

14.

$a^5, b^5, x^5, y^5, \dots$ - potencije jednakih eksponentata

$5 \cdot a^2 \rightarrow 5$ - koeficijent, a^2 - potencija
(druga potencija baze a)

Zbrajanje i oduzimanje potencija

$a^2 + b^2$ - ne može (različite baze)

$a^2 + a^3$ - ne može (različiti eksponenti)

$$5a^2 + 7a^2 = (5+7) \cdot a^2 = 12 \cdot a^2$$

$$9x^3 - 4x^3 + x^2 = (9-4) \cdot x^3 + x^2 = 5x^3 + x^2$$

$$4 \cdot 2^x + 5 \cdot 2^x = (4+5) \cdot 2^x = 9 \cdot 2^x$$

$$13b^4 + 5b^4 - 3b^4 =$$

$$8y^2 - 9y^3 + y^2 =$$

$$\frac{2}{3}a^4 - \frac{1}{6}a^4 + 2a^4 =$$

$$5a^7 - 3a^7 + 4a^8 - a^7 + 2a^8 =$$

$$x^5 - 2x^4 - 3x^5 - 7x^4 - 4x^5 + 5x^3 =$$

$$3a^3 + \frac{1}{2}a^2 - \frac{4}{3}a^3 - \frac{3}{4}a^2 - a^2 =$$

$$7 \cdot 3^x + 2 \cdot 3^x - 3^x =$$

$$6 \cdot 4^x - 5^x + 7 \cdot 4^x - 2 \cdot 5^x =$$

Računanje s potencijama jednakih baza

15.

<u>Množenje</u>	<u>Dijeljenje</u>	<u>Potenciranje</u>
$a^n \cdot a^m = a^{n+m}$	$a^n : a^m = a^{n-m}$	$(a^n)^m = a^{n \cdot m}$
$a^2 \cdot a^3 = a^{2+3} = a^5$	$b^7 : b^3 = b^{7-3} = b^4$	$(x^7)^5 = x^{7 \cdot 5} = x^{35}$
$x^5 \cdot y^2 \cdot x \cdot y^7 = x^6 \cdot y^9$	$(a^3 \cdot b^5) : (a \cdot b^2) = a^2 \cdot b^3$	$(3^x)^2 = 3^{2x} = (3^2)^x = 9^x$
$b^7 \cdot b^4 =$	$\left\{ \frac{a^3 b^5}{a^1 b^2} = a^{3-1} \cdot b^{5-2} \right\}$	$(a^5)^2 =$
$y^5 \cdot y =$	$x^9 : x^2 =$	$(b^4)^3 =$
$c^7 \cdot d^2 \cdot c \cdot d^4 =$	$a^7 : a =$	$(x^3)^5 : x^7 =$
$a^8 \cdot a^3 =$	$x^5 : x^2 \cdot x^3 =$	$(y^2)^4 \cdot y =$
$x^4 \cdot y^3 \cdot x^3 \cdot y^5 =$	$(b^7 \cdot c^5) : (b^2 \cdot c) =$	$(a^7)^4 : a^6 \cdot a =$
$a^3 b c \cdot a^4 b^5 c^7 =$	$(x^4 y^9) : (x \cdot y^3) =$	$(5^x)^2 \cdot 5^4 : 5^x =$

Potenciranje umnoška i kvocijenta

$(a \cdot b)^n = a^n \cdot b^n$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ ili $(a:b)^n = a^n : b^n$
$(5 \cdot x)^2 = 5^2 \cdot x^2 = 25x^2$, $(2^x)^3 = 2^{3x}$	$\left(\frac{2}{a}\right)^3 = \frac{2^3}{a^3} = \frac{8}{a^3}$
$(4a^5)^2 = \left(\frac{9}{8}a\right) =$	$\left(\frac{x^4}{y}\right)^5 = \left(\frac{3x^4}{2y^6}\right)^3 =$
$(2b^4)^3 = \left(\frac{3}{4}x^5\right)^3 =$	$\left(\frac{5a}{3b^4}\right)^2 = \left(\frac{2b^7}{c^8}\right)^4 =$
$(3x^2)^4 = \left(\frac{9}{10}c \cdot d\right)^2 =$	
$\left(\frac{7}{8}a^4 b^3\right)^2 = (5^x)^2 =$	

Računanje s potencijama jednakih eksponentata

5.

Množenje

$$a^n \cdot b^n = (a \cdot b)^n$$

$$2^6 \cdot 5^6 = (2 \cdot 5)^6 = 10^6 = 1\,000\,000$$

$$3^4 \cdot 2^4 =$$

$$4^3 \cdot \left(\frac{3}{2}\right)^3 =$$

$$5^4 \cdot 2^4 =$$

$$15^x \cdot 2^x =$$

$$3^a \cdot 13^a =$$

Dijeljenje

$$a^n : b^n = (a : b)^n$$

$$12^5 : 6^5 = (12 : 6)^5 = 2^5 = 32$$

$$8^6 : 4^6 =$$

$$10^4 : \left(\frac{5}{2}\right)^4 =$$

$$16^x : 8^x \cdot 3^x =$$

$$12^a \cdot 5^a : 30^a =$$

$$4^7 \cdot 3^7 : 6^7 =$$

Napiši kao potenciju (najmanje baze i eksponenta različitog od 1 ili -1)

$$8a^6 = 2^3 \cdot a^{3 \cdot 2} = 2^3 \cdot (a^2)^3 = (2a^2)^3$$

$$27 \cdot 9^x = 3^3 \cdot (3^2)^x = 3^3 \cdot 3^{2x} = 3^{3+2x}$$

$$\frac{16x^4}{81} =$$

$$\frac{27b^3}{8} =$$

$$32 \cdot 8^x =$$

$$25^x \cdot 125 =$$

$$2 \cdot 3^a + 3^a =$$

$$7 \cdot 5^x + 3 \cdot 5^{x+1} + 3 \cdot 5^x =$$

Računanje u skupu \mathbb{R}

[7.]

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad \text{ZAKON DISTRIBUTIVNOSTI}$$

Zadatak Izračunaj:

$$1.) \quad 6 \cdot (7x+5) = 6 \cdot 7x + 6 \cdot 5 = 42x + 30$$

$$5 \cdot (2a+3) =$$

$$4 \cdot (9b-8) =$$

$$a \cdot (4a-b) = a \cdot 4a - a \cdot b = 4a^2 - ab$$

$$x \cdot (2x-y) =$$

$$b \cdot (6b+7c) =$$

$$2b^3 \cdot (3b - 4b^2 + 5) = 2b^3 \cdot 3b - 2b^3 \cdot 4b^2 + 2b^3 \cdot 5 = \\ = 6b^4 - 8b^5 + 10b^3$$

$$9x^5 \cdot (2x^2 - 3x + 4) =$$

$$8a^4 \cdot (3a^5 + 2a^2 - 7) =$$

$$4x^2y^3 \cdot (5xy^4 - 7x^5y^2) = 4x^2y^3 \cdot 5xy^4 - 4x^2y^3 \cdot 7x^5y^2 = \\ = 20x^3y^7 - 28x^7y^5$$

$$3ab^4 \cdot (2a^5b^3 + 8a^3b) =$$

$$5x^4y^5 \cdot (6x^2y^3 - 7xy^8) =$$

$$2.) \quad (x+3) \cdot (x-7) = x \cdot (x-7) + 3 \cdot (x-7) = \quad \boxed{3.}$$
$$= \underbrace{x^2 - 7x + 3x - 21}_{=} = x^2 - 4x - 21$$

$$(a+2) \cdot (a+5) =$$

$$(b+4) \cdot (b-9) =$$

$$(x-6) \cdot (x+8) =$$

$$(y-3) \cdot (y-1) =$$

$$(2a-3b) \cdot (4a-5b) = 2a \cdot (4a-5b) - 3b \cdot (4a-5b) =$$
$$= 8a^2 - \underbrace{10ab - 12ab}_{=} + 15b^2 = 8a^2 - 22ab + 15b^2$$

$$(4x-5y) \cdot (2x-7y) =$$

$$(9a+4b) \cdot (3a-b) =$$

$$(8x^2-3) \cdot (2x-5) =$$

$$(7b^2+2) \cdot (3b^2-b-4) =$$

$$(2x^3+3x-5) \cdot (x^2-4x-1) =$$

3.) $\left\{ \begin{array}{l} (a+b) \cdot (a+b) = (a+b)^2 \Rightarrow (1+11)^2 = 1^2 + 2 \cdot 1 \cdot 11 + 11^2 \quad |9. \\ (a-b) \cdot (a-b) = (a-b)^2 \Rightarrow (1-11)^2 = 1^2 - 2 \cdot 1 \cdot 11 + 11^2 \\ a-b = -(b-a) \\ (a-b)^2 = (b-a)^2 \\ -a-b = -(a+b) \\ (-a-b)^2 = (a+b)^2 \end{array} \right.$

$$(5c - 1)^2 = (5c)^2 - 2 \cdot 5c \cdot 1 + 1^2 = 25c^2 - 10c + 1$$

$$(2x + 3y)^2 = (2x)^2 + 2 \cdot 2x \cdot 3y + (3y)^2 = 4x^2 + 12xy + 9y^2$$

$$(7a - 4b)^2 = (7a)^2 - 2 \cdot 7a \cdot 4b + (4b)^2 = 49a^2 - 56ab + 16b^2$$

$$(5x + 6)^2 =$$

$$(8a + b)^2 =$$

$$(2b - 9c)^2 =$$

$$\left(-\frac{3}{4}x + 2y\right)^2 =$$

$$\left(-6a - \frac{1}{2}b\right)^2 =$$

$$(7x^2 + 3y^4)^2 =$$

$$(9a^3 - 5ab)^2 =$$

$$(8xy^4 + 3x^5y^3)^2 =$$

$$(-4a^5 + 3ab)^2 =$$

$$(-2x^7 - 3y^5)^2 =$$

$$5) \begin{cases} (a+b) \cdot (a^2 - a \cdot b + b^2) = a^3 + b^3 & \text{zbroy' kubova} \\ (a-b) \cdot (a^2 + a \cdot b + b^2) = a^3 - b^3 & \text{razlika kubova} \end{cases} \quad \boxed{10}$$

$$(x+5) \cdot (x^2 - 5x + 25) = x^3 + 5^3 = x^3 + 125 \quad (\text{ili pomnoziti zagrada})$$

$$(y+3) \cdot (y^2 - 3y + 9) =$$

$$(2a+1) \cdot (4a^2 - 2a + 1) =$$

$$(4x+y) \cdot (16x^2 - 4xy + y^2) =$$

$$(y-3) \cdot (y^2 + 3y + 9) = y^3 - 3^3 = y^3 - 27$$

$$(x-4) \cdot (x^2 + 4x + 16) =$$

$$(a-2b) \cdot (a^2 - 2ab + 4b^2) =$$

$$(5b-3c) \cdot (25b^2 - 15bc + 9c^2) =$$

$$4.) \begin{cases} (a-b) \cdot (a+b) = a^2 - b^2 & \text{razlika kvadrata} \end{cases}$$

$$(5x-4) \cdot (5x+4) = (5x)^2 - 4^2 = 25x^2 - 16$$

$$(7a-b) \cdot (7a+b) =$$

$$(9y+2) \cdot (9y-2) =$$

$$\left(\frac{4}{3}x+y\right) \cdot \left(\frac{4}{3}x-y\right) =$$

$$(8a^2-b) \cdot (8a^2+b) =$$

$$(3x^4+2y^3) \cdot (3x^4-2y^3) =$$

11.

$$6.) \begin{cases} (1+11)^3 = 1^3 + 3 \cdot 1^2 \cdot 11 + 3 \cdot 1 \cdot 11^2 + 11^3 & \text{kub zbroja} \\ (1-11)^3 = 1^3 - 3 \cdot 1^2 \cdot 11 + 3 \cdot 1 \cdot 11^2 - 11^3 & \text{kub razlike} \end{cases}$$

$$(x+2)^3 = x^3 + 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 + 2^3 = x^3 + 6x^2 + 12x + 8$$

$$(2a-1)^3 = (2a)^3 - \underbrace{3 \cdot (2a)^2 \cdot 1}_{4a^2} + 3 \cdot 2a \cdot 1^2 - 1^3 = 8a^3 - 12a^2 + 6a - 1$$

$$(x+4)^3 =$$

$$(2y-3)^3 =$$

$$(3x+1)^3 =$$

$$(a-5)^3 =$$

$$(4b+3)^3 =$$

$$(5x-2y)^3 =$$

7.) Izračunaj !

$$1.) (2a-3b) \cdot (4a-b) - (7a-5b)^2 =$$

$$2.) (4x+1)^3 + (2x^2-3) \cdot (3x-2) =$$

$$3.) (2b-1)(4b^2+2b+1) - (7b-2)(b^2-3) =$$

$$4.) (9x^2-4) \cdot (9x^2+4) - (3x-4)^2 =$$

$$5.) (5x+3) \cdot (2x-7)^2 \cdot (5x-3) =$$

Rastavljanje na umnožak

1.) Izlučivanje zajedničkog faktora

$$8a + 4 = 4 \cdot 2a + 4 \cdot 1 = 4 \cdot (2a + 1)$$

$$10x - 15 =$$

$$28b^3 - 21b =$$

$$30a^5b + 20a^3b^2 - 10a^4b^3 =$$

2.) Rastavljanje na umnožak pomoću formula

a) dvočlani izrazi: $1^2 - 11^2 = (1 - 11) \cdot (1 + 11)$ - razlika kvadrata

$$1^3 - 11^3 = (1 - 11) \cdot (1^2 + 1 \cdot 11 + 11^2)$$
 - razlika kubova

$$1^3 + 11^3 = (1 + 11) \cdot (1^2 - 1 \cdot 11 + 11^2)$$
 - zbroj kubova

$$16a^2 - 25 = (4a)^2 - 5^2 = \left\{ \begin{array}{l} \text{razlika} \\ \text{kvadrata} \end{array} \right\} = (4a - 5) \cdot (4a + 5)$$

$$8x^3 - 1 = (2x)^3 - 1^3 = \left\{ \begin{array}{l} \text{razlika} \\ \text{kubova} \end{array} \right\} = (2x - 1) \cdot (2x)^2 + 2x \cdot 1 + 1^2 = \\ = (2x - 1) \cdot (4x^2 + 2x + 1)$$

$$y^3 + 27 = y^3 + 3^3 = \left\{ \begin{array}{l} \text{zbroj} \\ \text{kubova} \end{array} \right\} = (y + 3) \cdot (y^2 - y \cdot 3 + 3^2) = (y + 3) \cdot (y^2 - 3y + 9)$$

$$81x^2 - 4y^2 =$$

$$27a^3 - 8 =$$

$$\frac{1}{8}x^3 + 1 =$$

$$100a^2 - 49b^2 =$$

$$a^6 - 8 =$$

$$9x^4 - \frac{1}{4} =$$

$$45a^3 - 20a = 5a \cdot (9a^2 - 4) = 5a \cdot [(3a)^2 - 2^2] = 5a \cdot (3a - 2) \cdot (3a + 2)$$

$$72x^3y^3 - 2xy^2 =$$

$$3a^4b^2 - 24ab^5 =$$

b) tročlani izrazi : $1^2 - 2 \cdot 1 \cdot 11 + 11^2 = (1-11)^2$ - kvadrat razlike
 $1^2 + 2 \cdot 1 \cdot 11 + 11^2 = (1+11)^2$ - kvadrat zbroja

$$25a^2 - 10a + 1 = (5a)^2 - 2 \cdot 5a \cdot 1 + 1^2 = (5a-1)^2$$

$$4x^2 + 28x + 49 = (2x)^2 + 2 \cdot 2x \cdot 7 + 7^2 = (2x+7)^2$$

$$9b^2 + 6b + 1 =$$

$$x^2 - 16b + 64 =$$

$$16b^2 - 8b + 1 =$$

$$c^2 + 12c + 36 =$$

$$4a^2 + 20ab + 25b^2 =$$

$$\frac{12x^3}{3 \cdot 4} - \frac{36x^2y}{3 \cdot 12} + \frac{27xy^2}{3 \cdot 9} = 3x \cdot \left(\frac{4x^2}{(2x)^2} - \frac{12xy}{2 \cdot 2x \cdot 3y} + \frac{9y^2}{(3y)^2} \right) = 3x \cdot (2x-3y)^2$$

$$18b^3c + 48b^2c^2 + 32bc^3 =$$

$$49x^7 - 28x^5 + 4x^3 =$$

c) četveročlani izrazi ; $1^3 + 3 \cdot 1^2 \cdot 11 + 3 \cdot 1 \cdot 11^2 + 11^3 = (1+11)^3$ kub zbroja

$1^3 - 3 \cdot 1^2 \cdot 11 + 3 \cdot 1 \cdot 11^2 - 11^3 = (1-11)^3$ kub razlike

$$\frac{x^3}{x^3 \Rightarrow 1=x} + 9x^2 + 27x + \frac{27}{3^3 \Rightarrow 11=3} = x^3 + 3 \cdot x^2 \cdot 3 + 3 \cdot x \cdot 3^2 + 3^3 = (x+3)^3$$

$$8x^3 + 12x^2 + 6x + 1 =$$

$$a^3 - 6a^2b + 12ab^2 - 8b^3 =$$

$$y^3 + 6y^2 + 12y + 8 =$$

3.) Rastavljajući na umnožak grupiranjem članova

$$\begin{aligned}x^2 + ax + 5x + 5a &= (x^2 + ax) + (5x + 5a) = \\ &= x \cdot (x+a) + 5 \cdot (x+a) = (x+a) \cdot (x+5)\end{aligned}$$

$$\frac{2a^2 - 2ab - a + b}{2a \cdot (a-b) \quad -1(a-b)} = (a-b) \cdot (2a-1)$$

$$2x^2 + xy + 6x + 3y =$$

$$4a^2 + 8ab - 3a - 6b =$$

$$10b^2 - 15bc - 4b + 6c =$$

$$xy - 4y + 3ax - 12a =$$

$$x^2 - 6x + 9 - y^2 = \begin{cases} \rightarrow x \cdot (x-6) + (3-y)(3+y) ? \\ \rightarrow x^2 + 9 - 6x - y^2 ? \\ \rightarrow x^2 - y^2 - 6x + 9 ? \end{cases}$$

$$\checkmark = (x^2 - 6x + 9) - y^2 = (x-3)^2 - y^2 = (x-3-y)(x-3+y)$$

$$x^3 + x^2 - x - 1 = x^2 \cdot (x+1) - 1 \cdot (x+1) = (x+1) \cdot \underbrace{(x^2 - 1)}_{!} = (x+1)(x-1)(x^2+x+1)$$

4.) Rastavljajući na umnožak prikazivanjem jednog člana kao zbroj (kvadrati trinoma!)

$$\begin{aligned}a^2 + 7a + 12 &= (\text{nije potpun kvadrat}) = \underbrace{a^2 + 3a} + \underbrace{4a + 12} = \\ \left\{ \begin{array}{l} 7 = 3 + 4 \text{ i } 12 = 3 \cdot 4 \text{ ! } \Rightarrow 7a = 3a + 4a \end{array} \right\} &= a \cdot (a+3) + 4 \cdot (a+3) = \\ &= (a+3) \cdot (a+4)\end{aligned}$$

$$b^2 + 6a + 5 =$$

$$x^2 - 5a + 6 =$$

$$y^2 + 2y - 8 =$$

$$a^2 - 7a - 8 =$$